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A FACTOR ANALYSIS OF PROJECT TALENT TESTS AND FOUR OTHER TEST BATTERIES

Edward E. Cureton

American Institutes for Research
and
School of Education, University of Pittsburgh

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Major Project TALENT Publications

Flanagan, J. C., Dailey, J. T., Shaycoft, Marion F., Orr, D. B., Gorham, W. A., & Goldberg, I. *Designing the study.* (Technical report to the U. S. Office of Education, Cooperative Research Project No. 635.) Washington, D. C.: Project TALENT Office, Univer. of Pittsburgh, 1960.

Flanagan, J. C., Dailey, J. T., Shaycoft, Marion F., Gorham, W. A., Orr, D. B., & Goldbert, I. *The talents of American youth.* Vol. I. *Design for a study of American Youth.* Boston: Houghton Mifflin, 1962.

Flanagan, J. C., Dailey, J. T., Shaycoft, Marion F., Orr, D. B., & Goldberg, I. *Studies of the American high school.* (Final report to the U. S. Office of Education, Cooperative Research Project No. 226.) Washington, D. C.: Project TALENT Office, Univer. of Pittsburgh, 1962.

Shaycoft, Marion F., Dailey, J. T., Orr, D. B., Neyman, C. A., Jr., & Sherman, S. E. *Studies of a complete age group - Age 15.* (Final report to the U. S. Office of Education, Cooperative Research Project No. 635.) Pittsburgh: Project TALENT Office, Univer. of Pittsburgh, 1963.

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Flanagan, J. C., Cooley, W. W., Lohnes, P. R., Schoenfeldt, L. F., Holdeman, R. W., Combs, Janet, & Becker, Susan J. *Project TALENT one-year follow-up studies.* (Final report to the U. S. Office of Education, Cooperative Research Project No. 2333.) Pittsburgh: Project TALENT Office, Univer. of Pittsburgh, 1966.

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Hall, C. E. *Three papers in multivariate analysis.* (Interim report 2 to the U. S. Office of Education, Cooperative Research Project No. 3051.) Pittsburgh: Project TALENT Office, American Institutes for Research and Univer. of Pittsburgh, 1967.

Shaycoft, Marion F. *The high school years: Growth in cognitive skills.* (Interim report 3 to the U. S. Office of Education, Cooperative Research Project No. 3051.) Pittsburgh: Project TALENT Office, American Institutes for Research and Univer. of Pittsburgh, 1967.

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A FACTOR ANALYSIS OF PROJECT TALENT TESTS

AND FOUR OTHER TEST BATTERIES

Edward E. Cureton
University of Tennessee

Interim Report 4
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and

School of Education, University of Pittsburgh

1968

PREFACE

This study reports, for a group of 257 boys and a group of 286 girls, factor analyses of 43 Project TALENT aptitude and information tests together with 48 tests from three multiple-aptitude batteries and one high school achievement battery: the Flanagan Aptitude Classification Tests, the Differential Aptitude Tests, the General Aptitude Test Battery, and the Essential High School Content Battery. The subjects were high school juniors when the Project TALENT tests were administered in the spring of 1960, and seniors when the other tests were administered the following fall. All of them came from the rural and suburban areas of Knox County, Tennessee.

At least two previous Project TALENT reports include factor analyses, each by a different procedure. The results of this study are compared with the results of these two previous studies, and some general conclusions are drawn. The methods used in this study differ from those used in both of the others, so the first chapter deals with methodology.

I am deeply grateful to the many people who helped make this study possible. Dr. Mildred E. Doyle, Superintendent of Schools for Knox County, approved the project and obtained the cooperation of the county high schools. Miss Oriana Howley, Director of Guidance, made all the administrative arrangements. Special thanks are due to the principals, counselors and teachers of the Knox County high schools: every one of them responded magnificently in rearranging schedules and administering several large batteries of tests. The Essential High School Content battery was administered and scored as a part of the regular county testing program, and I am indebted to Miss D. Jean Reynolds of the State Testing Bureau for separate scoring of the subtests of the English test.

The American Institutes for Research lent reusable booklets, donated hand-scored booklets and answer sheets, and did all the scoring

for the Flanagan Aptitude Classification Tests. For this I am indebted to Dr. John C. Flanagan.

Thanks are due to the Psychological Corporation, and especially to Dr. George K. Bennett and Dr. Alexander G. Wesman, for donating the test booklets and answer sheets for the Differential Aptitude Tests and for scoring all the answer sheets.

I am indebted particularly to Mr. Frank P. Early of the State Department of Employment Security, and Mr. Fred W. Vance of its Knoxville office, not only for supplying all necessary materials for the General Aptitude Test Battery, including the apparatus tests, but for having all the examining and scoring done by their Knoxville staff.

Thanks are due to the University of Tennessee Computing Center, and in a very special sense to Mr. Richard Durfee, programmer. The factor-analytic procedures described in Chapter I were developed over a period of years with Mr. Durfee's help, and many of the opinions expressed there resulted from experience in using other procedures which were later modified or discarded.

Finally I am indebted to my wife, Dr. Louise W. Cureton, not only for encouragement and assistance throughout the study, but also in her capacity as Project TALENT Regional Coordinator for East Tennessee, for general supervision of all the testing and for liaison with the main office of Project TALENT as the study progressed.

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Chapter One

METHODOLOGY OF FACTOR ANALYSIS

Many years ago I wrote (Cureton, 1939), "Factor theory may be defined as mathematical rationalization ... Factor-analysts possess fixed ideas and also compulsions. The fixed ideas, after a sufficient number of rotations, become theories regarding the nature of mind and personality. The compulsions lead to the development of mathematical systems of analysis."

I see no reason now to repudiate or even modify this statement. But in the years since 1939 my own compulsions have become hardened and organized into a system of factor analysis. To me this is clearly the one best system. It is equally clear that practically no one else will agree with me. This chapter, then, not only describes a system of factor analysis, but defends the prejudices and compulsions on which the system is based.

We start with a correlation matrix. Only a few people will claim at this point that we should start instead with a variance-covariance matrix. There is fairly good agreement that the arbitrary metrics of aptitude and information tests had best be replaced by standard scores.

Systems of dimensional analysis may be subdivided initially into two main categories: component analysis and factor analysis (or more exactly common-factor analysis, for those who prefer "factor analysis" to "dimensional analysis" as the generic term). Component analysis is a legitimate multivariate method, concerned with the analysis of the total variance of a variance-covariance or correlation matrix, but it is not common-factor analysis. It has the advantage that the component scores of individuals can be computed, whereas their factor scores can only be estimated by regression or approximated by still cruder methods. Factor analysis, on the other hand, is concerned with the analysis of common variance. The unique variance of each variable is merely computed, recorded, and then usually forgotten, because it is not the variance of interest.

There are three main methods of factor analysis, and there could be four. They are defined by the way in which we regard the set of subjects and the set of variables. Each set may be regarded either as a finite population or as a sample from a larger (conceptually infinite) population. We then have the following table:

	Subjects	
Variables	Sample	Population
Sample		Alpha factor analysis
Population	Classical factor analysis	Image-covariance analysis

The upper left cell is blank because no one has yet devised a system based on the assumption that both the subjects and the variables are samples.

For the present study I choose classical factor analysis. The subjects are regarded as a sample of American high school juniors and seniors, albeit a somewhat biased sample. The tests are certainly not a random sample of all possible aptitude and information tests. They were carefully chosen to cover only certain particular regions within this domain. I should hope to generalize the results of this study, so far as biased sampling may permit, to high school juniors and seniors in general. The tests, however, I consider a finite population, and have no intention of generalizing to other tests unless the latter are very similar to those treated in this study.

There are a dozen-odd methods for performing a classical factor analysis. The diagonal or square-root method, all the grouping methods, and all variants of the centroid method can be dismissed at once. All of them are approximations to better methods. With the advent of the computer age, they can all be considered obsolete. This leaves only

the principal-axes method, and the maximum-likelihood method with its presently preferred variant, the canonical method. I choose the principal-axes method, and here a real defense appears necessary.

First, since I propose to rotate the initial factor matrix to simple structure, I am unimpressed by the scale-unit-invariance properties of the canonical method. This, however, is not an objection, but merely a slightly weak rebuttal. My real objection is to any method which employs large iterations (successive re-factorings) to reach communalities which are exact for the sample and the number of factors retained. The mathematical rank of the off-diagonal elements of a sample correlation matrix is less than its order with probability zero. Small useless factors are generated not only by the sampling errors, but also by chance correlations among the form-associated errors of measurement, and by both chance and non-chance correlations among the time-associated errors of measurement which arise because tests are administered serially rather than simultaneously. These factors, though small, are real in the sample, and the largest of them are larger than the smallest of the substantive factors.

The effective communalities of a correlation matrix are those obtained as the sums of squares of the loadings on the initial factors which can be meaningfully rotated. From the argument just above, it is clear that the number of such factors is always less than the number of real factors. If we stop factoring at this point and use large iterations to find the corresponding "exact" communalities, there is genuine danger of a Heywood case, with one "exact" communality greater than unity; and a very much greater danger of what I shall term a quasi-Heywood case, with one or more communalities greater than the corresponding test reliabilities.

When we factor a sample matrix, the assorted errors are distributed more or less randomly over subjects and tests, but they are not distributed uniformly. When we fit the factors to the sample data, in consequence, the dispersion of the sample communalities is inflated as

compared to the dispersion of the true communalities in the population. Exact fitting by repeated large iterations increases this dispersion still further: hence the danger of a Heywood or quasi-Heywood case. This effect is analogous to an effect observed in multiple regression analysis. If we physically draw samples from a population having known regression coefficients, the sample regression coefficients almost always show greater dispersion than do those of the population. The least-squares fitting procedure, when applied to the sample, fits the sampling errors (and the errors of measurement if any), as well as the true relationships.

Even in the sample, large iterations apply properly to the determination of the real communalities rather than the effective communalities. If we retain enough factors initially to yield the real sample communalities, thus minimizing the danger of a Heywood or quasi-Heywood case, we will have to get rid of some of them in the rotational procedure, and determination of these exact real communalities is pointless. The communalities of interest are the effective communalities, not the real sample communalities, and we must simply recognize that the corresponding unique factors always include common factors too small and mixed up with assorted errors to be rotated meaningfully.

It seems to me, then, that the advantages of the canonical method are largely illusory, and that the method of choice for initial factoring should be the principal-axes method, with at most one or two large iterations to insure that the number of factors retained has not been biased by the errors in the initial communality estimates. There remain then only problems of detail in the use of the principal-axes method, and the twin perennial problems of communality estimation and the number of factors.

Initial Communality Estimation

The squared multiple correlations (SMC's) are lower bounds to the real communalities, but not necessarily to the effective communalities. The errors of measurement, on the other hand, do certainly attenuate the intercorrelations. Individually, the $|r|_{max}$ values -- the absolute

values of the numerically highest correlations in the several columns -- are not very good estimates of the corresponding effective communalities. Some of them are overestimates and some of them are underestimates. Their sum, on the other hand, appears both empirically and in theory to yield a fairly good estimate of the effective trace: the sum of the effective communalities. When in error it is somewhat more likely to yield an overestimate than an underestimate.

With small batteries of reliable tests, in which each rotated factor has only a small number of non-zero loadings, the SMC's often yield gross underestimates of the effective trace, while the $|r|_{max}$ values still yield good estimates. And with large batteries of unreliable tests, especially when the tests are single items and are five or six times as numerous as the useful factors, most useful factors may have quite a number of non-zero loadings. In such cases, since the multiple correlation procedure fits the errors as well as the real relationships in the sample, the SMC's may yield an overestimate of the effective trace. In these situations the errors of measurement attenuate the $|r|_{max}$ values, and their sum is still likely to give a fairly good estimate.

The SMC's, each of which is based on all the intercorrelations, tend to go up and down all together: they tend all to yield overestimates or all to yield underestimates of the corresponding effective communalities. Since each SMC is in fact the "finite communality," the true communalities and the effective communalities should be quite closely proportional to the SMC's.

From these considerations we arrive at a formula for initial estimates for the effective communalities: they should be proportional to the SMC's, but with sum equal to the sum of the $|r|_{max}$ values, or

$$\hat{h}_i^2 = SMC_i \frac{\Sigma |r|_{max}}{\Sigma (SMC)}, \qquad (1)$$

the sums going from $i = 1, \ldots, n$, the number of variables.

There is a tacit assumption in the preceding argument that the $\hat{h}_i^2$ should be estimates of the effective communalities rather than of the real sample communalities. This assumption is open to further argument.

If R^* is a correlation matrix with estimated communalities in the diagonal, it is a Gramian matrix if there exists a matrix F such that

$$R^* = FF' \tag{2}$$

If the mathematical rank of R^* is $m<n$, F will have only m non-zero columns, and R^* will have m positive eigenvalues and $n-m$ eigenvalues which are exactly zero. A Gramian matrix does not have negative eigenvalues.

Now (2) has been termed the fundamental equation of factor analysis, and it is if we consider only a population of subjectsand a correlation matrix whose mathematical rank is $m<n$. In view of these considerations, some factor analysts demand that the sample correlation matrix be Gramian or almost Gramian. They are thus led to use initial communality estimates which are gross overestimates of the final effective communalities: estimates of the real sample communalities or even unities. In fact since, as noted previously, the mathematical rank of a sample correlation matrix beset with both sampling errors and two kinds of errors of measurement in every variable cannot be expected to be less than its order, the only way to be certain it will be strictly Gramian is to use unities as communality estimates.

When we deal with a correlation matrix based on a finite sample, with variables all of which have assorted errors of measurement and sampling errors, the fundamental equation of factor analysis should be

$$R^* = FF' + \Delta, \tag{3}$$

where Δ is the residual correlation matrix after the last factor retained. It seems to me that the initial communality estimates should be so chosen as to minimize Δ; i.e., since the principal-axes procedure

is a least-squares procedure, they should be so chosen that $\Sigma\Sigma\delta_{ij}^2$ is a minimum. This would imply that the sum of the last n-m eigenvalues of R* should equal zero, and negates incidentally the proposal that they should be so chosen that m will be the number of factors corresponding to positive eigenvalues of R*. The factors not retained are assumed to be "error factors," with eigenvalues differing from zero only by chance, and hence equally likely to be positive or negative.

The matrix Δ should have diagonal elements of about the same order of magnitude as its off-diagonal elements, since all are assumed due to error. But when, by repeated large iterations, we "stabilize" the communalities, the diagonal elements of Δ all become exactly zero. This is not in accord with the assumption that Δ is an error matrix throughout, and the procedure of forcing it to have all diagonal elements exactly zero is another way of showing how the danger of a Heywood or quasi-Heywood case is increased.

The correlation matrix R* should certainly be "statistically Gramian." For the factors retained, every eigenvalue should be substantially positive, and every diagonal element of every residual correlation matrix should be positive.

Ideally the diagonal elements of Δ should sum to zero, but in practice initial communality estimation is not good enough to permit the diagonal of the residual correlation matrix for the last factor retained to be all-positive while the diagonal of Δ is half positive and half negative. In this situation I lean just slightly toward the Gramian viewpoint. As noted previously, an initial trace equal to $\Sigma|r|max$ is somewhat more likely to overestimate than to underestimate the effective trace, and in every residual correlation matrix I replace any diagonal element by one-half the mean of the absolute values of the off-diagonal elements in the column if the latter is algebraically larger. But this is a much smaller correction than the one proposed many years ago by Thurstone: to replace all diagonal elements by the corresponding $|r|max$ values in every residual matrix. Note that while $\Sigma|r|max$ is a good estimate of the initial trace, it becomes a progressively

worse overestimate of each residual trace; the ratio of residual trace to $\Sigma|r|max$ should decrease for every successive residual matrix until for Δ it becomes zero. With initial estimates as good as those given by (1), diagonal residuals can be used throughout unless one of them gets too close to zero or becomes negative, and the rule of thumb described above seems in practice to be all we need to do to take care of such situations.

The net effect of the slight overestimation of effective trace given by (1), and the procedure by which all diagonal elements are forced to remain positive and at least half as large as the mean absolute value of the off-diagonal elements in the column, is that the computed communalities for the factors retained have a sum which is usually slightly less than the initial trace.

Details of Initial Factoring

The fact that the diagonal elements of each residual matrix may have to be adjusted dictates successive rather than simultaneous extraction of the principal-axes factors. Here most factor-analysts use Hotelling's scaling factor: after each multiplication of the correlation matrix by a vector of trial factor loadings, the product vector is re-scaled by dividing each of its elements by the largest. With this method, convergence to both the eigenvalue and the vector of factor loadings must be complete if the next residual matrix is to be of rank exactly one less than that of the preceding matrix.

Horst (1961) describes an improved procedure, and this procedure is also described in somewhat more compact form in his book on factor analysis (Horst, 1965). If F_i is a vector of trial factor loadings, and R^* is a correlation matrix with estimated communalities in the diagonal, or a residual correlation matrix, the iteration formula is

$$F_{i+1} = \frac{R^*F_i}{\sqrt{F_i'(R^*F_i)}}, \qquad (4)$$

and the scaling factor is the reciprocal of the denominator. When sufficient convergence has taken place, say at the k-th iteration, F_k becomes one column of F, the principal-axes factor matrix, and the denominator is the corresponding eigenvalue. This procedure has two valuable features:

1) If the computations are terminated at any iteration after the first, $R^* - F_i F_i'$ will be of rank exactly one less than R^*.

2) At every iteration after the first, the denominator approaches the largest eigenvalue from below.

If two eigenvalues are close together, convergence of the denominator to the larger is much faster than is convergence of F_i to the true vector of factor loadings. But if we terminate the iterations early, the individual variables-variances lost in F_i will be picked up in F_{i+1}, with no harm resulting, since we will be rotating the final F anyhow. They can even be terminated when the approximation to E_i, the i-th eigenvalue, is less than E_{i+1}. In this case the next <u>computed</u> eigenvalue, E_{i+1}, will be larger than E_i, and this is still no cause for concern provided we resolve in advance that the number of factors retained will never be exactly i if $E_{i+1} > E_i$, or if the difference $E_i - E_{i+1}$ is very small. In practice, therefore, I terminate an iteration when the increase in the denominator of (4) from one iteration to the next does not exceed .0001.

For the first trial vector, F_o, several authors suggest the unit vector. For factors after the first, this is usually a poor starting point, for if a residual matrix is not reflected, the unit vector is almost orthogonal to the final position, and would be exactly orthogonal if the principal axis coincided exactly with the centroid. It happens occasionally, moreover, that the sum of all the elements of a residual matrix is negative. In this case the expression under the radical in (4) is negative, it has no positive square root, and the computer emits an error signal at the first iteration.

As a start, therefore, I use a zero-one vector with a unity in the position of the largest diagonal element of R^* and zeros everywhere else. Then R^*F_o is simply the column of R^* whose diagonal element is largest, and the denominator of (4) is the square root of this diagonal element. This first iteration is programmed directly rather than by use of (4).

On the Number of Factors

In harmony with the proposition that the initial communality estimates should be estimates of the effective trace, I try to retain precisely the number of factors that can be meaningfully rotated. Real artists at hand rotation can over-factor initially and then "residualize" the error factors. I find, however, that with existing programs for numerical and analytic rotation, computers seem to lack the necessary artistry.

There appears to be no one method for determining the number of factors which can be rotated meaningfully. Following Tryon, I shall term this the number of salient factors. Even if we had an exact test of statistical significance, and an agreed-upon rationale for selecting an α-level, the number of salient factors would not necessarily be the number of significant factors. With very large N, it might be less: I have seen significant factors (N = 1000) with no loading greater than .20. With small N, it might be more; there is a crude analogy here to the case in simple analysis of variance with many categories of one class, where the F-test shows insignificance but a multiple-comparison test shows high significance for one category. An insignificant factor can sometimes determine a doublet or triplet of quite high significance and interpretability.

A significance test, nevertheless, is useful as one criterion among several. The Bargmann test (Bargmann, 1957; Bargmann and Brown, 1961) seems to be the most useful of the significance tests so far proposed. Though derived on the assumption of maximum-likelihood factoring, it appears to work quite well with principal-axes factoring. The equation

is

$$\chi^2 = [N - \frac{(2n + 11)}{6} - m] [\sum^{n} \ell n(1 - h_m^2) - \ell n|I_m - F_m' R^{-1} F_m| - \ell n|R|] ;$$

$$DF = n(n - 1)/2 , \tag{5}$$

for N subjects, n variables, and m factors; with h_m^2 a computed communality for the first m factors of R^*, F_m the first m columns of the principal-axes factor matrix, and R the correlation matrix with unities in the diagonal. To use this formula we require the inverse and determinant of R, but the former is required for the computation of the SMC's anyhow, and if we compute it by the Gaussian elimination procedure with diagonal pivots, $|R|$ is simply the product of all the pivotal elements. The determinant $|I_m - F_m' R^{-1} F_m|$ is not as formidable to compute as it first appears. If we first over-factor to k factors, we compute just once the matrix $M = I_k - F_k' R^{-1} F_k$. Each successive determinant for $m = 1, 2, \ldots, k$ is then merely the product of the first m pivotal elements of a Gauss forward solution of the matrix M.

Since $DF = n(n - 1)/2$ is usually fairly large, I use the Fisher transformation,

$$x/\sigma = \sqrt{2\chi^2} - \sqrt{2DF - 1} , \tag{6}$$

and print out χ^2 and x/σ for $m = 1, 2, \ldots, k$.

For each m, the null hypothesis is that m factors are sufficient, so the number of significant factors is the smallest value of m for which x/σ implies insignificance. In application, it appears best to set the α-level quite high. I seldom regard a positive x/σ value as insignificant unless it is less than 1.00.

The most generally useful test for the number of salient factors seems to be the scree test (Cattell, 1966). To apply this test, the eigenvalues are listed in order of magnitude, and beside them a column of first differences. In clear cases, the differences become progressively

smaller, there is then one larger difference, and the remaining differences are all appreciably smaller. Thus for the classic Holzinger-Harman 24 psychological tests we have (Harman, 1960, p. 188),

Factor	Eigenvalue	Dif.	
1	7.629		
		5.981	
2	1.648		
		.480	the "slope"
3	1.168		
		.273	
4	.895		
		.496	
5	.399		
		.053	
6	.346		
		.079	
7	.267		the "scree"
		.017	
8	.250		
		.039	
9	.211		

The reversal in the difference column from .273 to .496, followed by differences all less than .100, clearly indicates the presence of four salient factors.

With less clear data, it may be advisable to plot the numerical values of the eigenvalues against their ranks (the factor numbers). If we then fit one curve to the slope and another to the scree, the two curves may show a discontinuity where they meet, even though there is no clear difference-reversal. But if one single smooth curve fits all the eigenvalues, the scree test gives equivocal results.

A third test for salience may be made simply by examining the over-factored initial factor matrix. We should almost always retain enough columns to include the highest loading in every row, and more generally to keep most of the higher loadings in every row.

There will still be doubtful cases, and here the only solution appears to be to rotate two or more different numbers of factors to see which number, after rotation, seems to yield the clearest interpretation.

Simple Structure

The simple-structure criterion appears to be the weakest rotational criterion so far proposed, and I prefer it for that reason. By "weakest," I mean that it imposes the fewest restrictions consistent with a unique solution. The hierarchical orthogonal solution is algebraically equivalent, however, and may be preferred if the additional information supplied by the higher-order factors is significant. The perfect bi-factor solution (with one general factor and non-overlapping group factors) is, apart from the fitting procedure, simply the hierarchical orthogonal solution for the special case of one second-order factor and first-order factors all in clusters about the primary axes.

A simple structure is defined by the bounding hyperplanes of the configuration of n test vectors in m-space. The rules given by Thurstone (1947, p. 335) represent merely a not-quite-perfect description of a rotated factor matrix (a V-matrix) of projections on the reference vectors orthogonal to the bounding hyperplanes. The hyperplanes should usually be significantly overdetermined, but in rare cases even this requirement can be relaxed for one or two factors if the number of variables is small. A non-bounding hyperplane, on the other hand, cannot be accepted as defining a simple-structure factor no matter how greatly it may be overdetermined.

So long as all factors are definitely determined, with most of them substantially overdetermined, it is not necessary that every test vector lie in at least one hyperplane. A test vector can be close to the first principal axis, with low non-zero loadings on all factors. Such a test vector is merely useless in helping to locate the bounding hyperplanes.

A bounding hyperplane is defined by a subset of the n test vectors, which have near-zero loadings on its reference vector. Hence no analytic function of all the entries in an F-matrix can define it exactly.

A configuration of test vectors may have outer edges which are either smooth or irregular. In the former case the near-zero loadings may vary only between ±.05. In the latter case the width of the hyperplane bounds

will be directly related to the amount of overdetermination. In such cases I often allow the bound to be as wide as ±.15 and occasionally ±.20, so long as no single test within the bounds could reasonably be interpreted as having anything significant in common with the tests which have high loadings and determine the interpretation of the factor. Thus I reject all "hyperplane-count" criteria of excellence of hyperplane fit, since they are based on arbitrary definitions of the hyperplane bounds (usually ±.10).

With real data, the effectively bounding hyperplanes will all be orthogonal with probability zero. "Orthogonal simple structure" therefore means merely "orthogonal approximation to simple structure."

Rotational Procedures

The rotational procedure I prefer rests not so much on prejudice (other than prejudice in favor of simple structure) as on laziness. I never resort to plotting if there are more than three salient factors. It is the exigencies of my system of rotation, rather than any inherent or defensible beliefs, that dictate a very determined effort to put the simple structure in the positive manifold.

The first step comes even before the start of the initial factoring. The correlation matrix is reflected until all column sums, exclusive of diagonal entries, are positive. The variables reflected are not re-reflected until the rotation is finished, if at all. Instead, the names of the reflected variables are reversed, either temporarily or permanently. Inversion and principal-axes factoring are performed on the reflected correlation matrix.

The second step comes when the F-matrix is determined. If any variable has a negative loading on the first principal axis it is reflected: the signs of all loadings in the given row of F are changed, and the name of the variable is reversed or re-reversed.

The first rotational step consists of an incomplete normal varimax rotation. First the rows of F are normalized. The varimax rotation

(Kaiser, 1958) then moves the axes to positions such that the variance of the factor loadings of all variables on all rotated factors is a maximum, subject to an orthogonality restriction. The normal varimax matrix is not denormalized, and the transformation matrix is not computed. For my purposes, the normal varimax rotation needs to be only a good enough orthogonal approximation to simple structure to insure that every varimax hyperplane will be closer to the corresponding simple-structure hyperplane than to any other bounding or non-bounding hyperplane. In my experience with its use, it is always at least this good.

The next aim for the positive manifold comes at this point. If the sum of any column of the normal varimax factor matrix is negative, all signs in that column are reversed.

The next step in the rotational procedure is a modified promax rotation (Hurley and Cattell, 1962; Hendrickson and White, 1964). A hypothesis matrix H is constructed from the normal varimax factor matrix by cubing each of its elements. If we cube a loading of .8, the result is .512; if we cube .3, we obtain .027. Thus each column of H looks much more like a column of a simple-structure matrix than does the column of the normal varimax factor matrix from which it was constructed, but the transformed loadings are still in the same order. In accordance with the notion of aiming for positive manifold, however, all negative loadings in the normal varimax factor matrix are replaced by zeros in the hypothesis matrix.

A procrustes rotation of F toward H then yields an approximation to the best least-squares fit to H that can be obtained by an oblique rotation of F. Corresponding to the basic rotational formula, $F\Lambda = V$, we set up the corresponding formula,

$$FL = H \ . \tag{7}$$

Here F and H are given, and solving for L,

$$L = (F'F)^{-1}F'H \ . \tag{8}$$

But if F is a principal-axes factor matrix $F'F$ is the diagonal matrix E of the first m eigenvalues of R, and E^{-1} is a diagonal matrix with diagonal elements which are the reciprocals of these eigenvalues. Then (8) becomes

$$L = E^{-1}F'H \ , \tag{9}$$

and L normalized by columns becomes a transformation matrix P, so that

$$FP = V \ , \tag{10}$$

and V is the promax approximation to the simple-structure factor matrix.

Limited experience suggests that the promax rotation yields just about as good an approximation to oblique simple structure as do any of the more complicated analytic oblique rotations.

This is the point at which most factor-analysts would "clean up" the structure by using plots. I use instead a modification of Thurstone's "Analytic" (really only partially analytic) single-hyperplane procedure (Thurstone, 1954). Each column of V is iterated separately, along with the corresponding column of P. Let V_o be one column of V, let P_o be the corresponding column of P, and let $[V_o]$ be the vector V_o with its elements rearranged in order of magnitude from highest positive to highest negative (or to lowest positive if there are no negative elements in V_o), with the original row-indices printed alongside the loadings.

Looking up and down $[V_o]$, a cutting point is selected below which the loadings will be taken provisionally to be near-zero. This level will usually be somewhere near +.10, but it should also be not appreciably lower than the point at which the sum of squares of the

positive elements below it is roughly equal to the sum of squares of the negative elements at the bottom. In addition, it should preferably come at a "gap" -- a point at which there is a larger-than-average difference between two adjacent loadings. All negative loadings are treated initially as near-zeros.

Now let A_o be a submatrix of F consisting of those rows of F whose row-numbers correspond to those of the presumed near-zero elements of $[V_o]$. Form the m by m matrix $A_o'A_o$. Then,

$$A_o'A_oU_o = P_o \quad \text{(solve for } U_o\text{)}, \tag{11}$$

$$U_o \text{ normalized} = P_1, \tag{12}$$

$$V_1 = FP_1, \tag{13}$$

$$[V_1] = V_1 \text{ rearranged} . \tag{14}$$

P_1 , V_1, and $[V_1]$ are the revised values at the end of the first iteration. A new cutting point is set, a new submatrix A_1 of F is thereby defined, and the second iteration is

$$A_1'A_1U_1 = P_1 \quad \text{(solve for } U_1\text{)}, \tag{15}$$

$$U_1 \text{ normalized} = P_2 , \tag{16}$$

$$V_2 = FP_2 , \tag{17}$$

$$[V_2] = V_2 \text{ rearranged} . \tag{18}$$

At about the second iteration, the largest one or a few negative elements of $[V_2]$ are examined. If they are larger than the largest positive near-zero, they are given <u>weights</u> of 2, 3, or more. We now have a weight vector, W_2, most of whose elements are unity, but with

one or a few which are larger. Then in place of (15)

$$\acute{A}_2\acute{W}_2\acute{A}_2U_2 = P_2 \text{ (solve for } U_2), \qquad (19)$$

and the rest of the third iteration proceeds as before. All of the weights used in one iteration must be used in all following iterations unless changed for cause. Very occasionally the one or two largest positive near-zero loadings may be weighted also, usually only if there is a fairly large gap above the largest.

The use of weights permits turning what is otherwise essentially a least-squares hyperplane-fitting procedure into a rough minimax procedure. The best hyperplane fit occurs when the highest positive and the highest negative near-zero loadings are almost equal and as small as possible.

If the one or two largest negative loadings in any $[V_k]$, are not substantially reduced by weighting them as much as 4 or 5, and especially if use of these or higher weights brings new variables to the top (non-zero region) of $[V_{k+1}]$, indicating a swing of the hyperplane toward a different factor, these one or two variables are removed from the near-zero list and we have a lower cutting point as well as an upper cutting point to define the next A-matrix. Such variables are then accepted as having <u>intrinsically negative</u> loadings on the factor in question.

Note that by aiming for the positive manifold, we have placed within the subspace bounded by the hyperplanes (including the hyperplane bounds defined by the near-zero loadings), every test-vector which can be so placed by reflection. If all the test vectors do not actually lie in the positive manifold, they do at least all lie on one side of the hyperplane orthogonal to the first principal axis. And this axis, with all coordinates positive, lies fairly close to the center of the positive manifold. If a test then has an intrinsically negative loading, its vector lies <u>outside</u> the subspace bounded by the hyperplanes, and the corresponding factor is intrinsically bipolar. The number of such test vectors must be small: if it were not, the hyperplane would not be effectively a

boundary of the configuration. Consistent aim for the positive manifold is necessary to assure the finding of the bounding hyperplanes by consideration of the signs of the loadings, without the use of plots.

The iterations for each factor are continued until all the near-zeros are as small as possible. They may be continued also to complete consistency, which occurs when every loading in $[V_{k+1}]$ which was treated as a near-zero in $[V_k]$ is numerically smaller than every other; i.e., when the next A-matrix would have the same rows identically as had the immediately preceding A-matrix. Complete consistency may be reached earlier than good minimax fit, but the reverse is likely to be the case unless there is a substantial gap between the highest positive near-zero loading and the lowest positive non-zero loading, and a similar situation exists at the negative end if the factor is intrinsically bipolar.

When the rotation is complete, each final V_k becomes one column of the simple-structure factor matrix V, and each P_k becomes the corresponding column of the transformation matrix Λ. At this point, any tests which were reflected in the factor matrix and/or the F-matrix can be re-reflected. All that is necessary is to re-reverse the names of these tests, and to change all signs in the corresponding rows of F and V. The transformation matrix Λ is not affected. This procedure leads to nominally bipolar factors, with test vectors some of whose termini lie below the hyperplane orthogonal to the first principal axis. An intrinsically negative loading may even become nominally positive if the corresponding test is re-reflected. Whether or not tests should be re-reflected is an issue of interpretation rather than of analysis.

Chapter Two

THE TESTS AND THE SAMPLES

The test battery included 91 tests: 43 from the Project TALENT battery and 48 from the other four batteries.

The Project TALENT tests included all those in Information I except the Screening test, all those in Information II which had at least nine items, and all of the other educational and aptitude tests. The Vocabulary scores from Information I and II were combined into one score, and the Hunting and Fishing scores (each based on five items) were also combined. These tests were given in March and April 1960 as a part of the national Project TALENT testing program.

The other tests were administered at various times during the fall of 1960, from late September to early December.

The Essential High School Content Battery (Form BM, 1950) was administered as a part of the regular fall high school testing program. For the Mathematics, Science, and Social Studies tests, only the total scores were used, but for the English test separate scores were recorded for the Reading, Vocabulary, Business Definitions, Use of References, Literature Acquaintance, Language Usage, Capitalization and Punctuation, and Spelling subtests.

The Flanagan Aptitude Classification Tests included 17 of the 19 tests of the 1957 edition: all except Precision and Coordination.

For the Differential Aptitude Tests (third edition, 1947, Form A), all tests were used, and for the Language Usage test, the Spelling and Sentences parts were scored separately.

All tests of the General Aptitude Test Battery (separate-answer-sheet Form A, B-1002A, 1952, and the apparatus tests) were used.

With the exception of the General Aptitude Test Battery, which was administered by personnel of the local office of the State Department of Employment Security, all tests were administered by the teachers, under the general direction of the Project TALENT Regional Coordinator and the Director of Guidance of the school system.

The students and teachers of ten county schools participated in the study. About half the students came from the suburban areas surrounding a city of about 120,000 population, and about half came from rural areas. About 1600 took the Project TALENT tests as high school juniors in late March and early April. Of these about 1500 took one or more of the other four tests the following fall.

A large number of students missed one or more test sessions, and in one school it was discovered that in one class the teacher had shortened the time limit for one test by almost one-half because the bus driver wouldn't wait.

The rosters of scores from the four non-TALENT batteries were sent to the Project TALENT office to be punched on IBM cards, transferred to tape, and merged with the Project TALENT scores. In the merging process more cases were lost because of inconsistent identification data on the many answer sheets and record forms of each student. Efforts were made by the regular Project TALENT staff to resolve as many of these inconsistencies as possible by hand sorting of answer sheets by school and subsequent correction of the cards, but with only limited success.

For the regular Project TALENT staff, this has been a peripheral study, to be pursued when work on the major studies permitted. In consequence, several years elapsed between the completion of the testing and the delivery to me of the final data tape.

When frequency distributions were prepared and examined, those of the Precision and Coordination subtests of the Flanagan Aptitude Classification Tests appeared so anomalous in comparison

to the others that some sort of error in either administration or scoring was strongly suspected. By this time the answer booklets were no longer available to check for possible scoring errors, so these two tests were deleted from the battery.

As a result of all the factors noted above, the sample was greatly reduced. In the interest of consistency it was decided to employ for factor analysis only those subjects for whom complete data were available. The total number of such subjects was 543: 257 boys and 286 girls. Since there are substantial sex differences on some of the tests, it had been decided at the outset to perform separate factor analyses for the boys and the girls.

Table I lists the 91 tests by title, gives a brief description wherever the nature of a test is not obvious from its title, and records a code symbol (for the Project TALENT tests) or a subtest number for each test, and also the maximum possible raw score. For most of the tests the maximum raw score is the number of items, but for a few multiple marking is used with variable credit per item, and in these cases the maximum score is greater than the number of item-exercises.

From Table I it may be seen that the maximum raw scores range from 9 to 150. There are, hence, large differences in the test consistencies (form-associated reliabilities), with corresponding large differences in the upper limits of the communalities. The elapsed time between the administration of one test and another varies from a minute or two (between tests administered serially from the same booklet) to over nine months, and the mean time interval separating the administration of the Project TALENT tests from the administration of the other tests is about seven months. Time-associated errors are therefore highly variable: correlations between tests administered months apart will be considerably attenuated in comparison with correlations between tests administered on the same day. The sizes of the error

factors in these data will therefore be larger than they would be if the tests were more nearly equal in reliability and were all administered in a few consecutive sessions. Hence we cannot expect to be able to extract as many substantive factors as we might under these latter conditions. The relatively modest sizes of the final samples (in relation to the number of variables) will impose further limits on the numbers of salient factors.

Table I. The Tests

Var. No.	Code or Subtest	Title of Test Project TALENT	Max. Score
1	R102 + R162	Vocabulary	30
2	R103	Literature (information)	24
3	R104	Music (information)	13
4	R105	Social Studies (information)	24
5	R106	Mathematics (information: verbal)	23
6	R107	Physical Science (information)	18
7	R108	Biological Science (information)	11
8	R109	Scientific Attitude	10
9	R110	Aeronautics and Space (information)	10
10	R111	Electricity and Electronics (information)	20
11	R112	Mechanics (information)	19
12	R113	Farming (information)	12
13	R114	Home Economics (information)	21
14	R115	Sports (information)	14
15	R131	Art (information)	12
16	R132	Law (information)	9
17	R133	Health (information)	9
18	R139	Accounting, Business, Sales (information)	10
19	R142	Bible (information)	15
20	R145 + R146	Hunting and Fishing (information)	10
21	R147	Outdoor Activities, Other (information)	9
22	R211	Memory for Sentences (memorize 40 short sentences. For 16, supply one missing word later)	16
23	R212	Memory for Words (Study 24 "Vlaznoor"-English pairs. For 24, recognize English equivalent later)	24
24	R220	Disguised Words (Recognize SURKL, e.g., as round)	30
25	R231	Spelling (Identify misspelled word if any from list of 4, plus "None of above")	16
26	R232	Capitalization (Paragraph all L.C. Mark Cap. or no Cap. for 33 words)	33
27	R233	Punctuation (Sixteen short sentences, not punctuated; 3 to 5 versions of one or two words; check version correctly punctuated. Also eleven "sentences" to be identified as incomplete, complete, or two sentences run together.)	27
28	R234	Usage (Sentence with missing word or phrase. Select best fill-in)	25
29	R235	Effective Expression (Same sentence in 3 or more versions. Select best)	12
30	R240	Word Functions in Sentences (Stem sentence and answer sentence. Select word in answer sentence which has same function as capitalized word in stem sentence)	24

Table I (continued)

Var. No.	Code or Subtest	Title of Test	Max. Score
		Project TALENT	
31	R250	Reading Comprehension (Paragraph and questions)	48
32	R260	Creativity (Practical problem stated. Examinee selects clever solution: answers give only first and last letters)	20
33	R270	Mechanical Reasoning (Like Bennett)	20
34	R281	Visualization in 2 Dimensions (Rotate key figure to match answer without turning over)	24
35	R282	Visualization in 3 Dimensions (Pattern and 5 fold-ups. Pick correct fold-up)	16
36	R290	Abstract Reasoning (2-way figure matrix. Select choice for missing element)	15
37	R311	Arithmetic Reasoning (verbal problem and 4 or 5 options)	16
38	R312	Introductory Mathematics (Advanced arithmetic and elementary algebra)	24
39	R333	Advanced Mathematics (Advanced algebra, geometry, and trigonometry)	14
40	F410	Arithmetic Computation (add, subtract, multiply, and divide whole numbers)	72
41	F420	Table Reading (Two-argument table: dollar entries)	72
42	F430	Clerical Checking (Like Minnesota name checking)	74
43	F440	Object Inspection (Identical forms: one different)	40
		Flanagan Aptitude Classification Tests	
44	1	Inspection (Identical forms: one different)	80
45	2	Mechanics (Pictures, each with several questions)	30
46	3	Tables (Two-argument tables: RPM and Name entires)	120
47	4	Reasoning (Verbal Problem to formula or answer)	24
48	5	Vocabulary	60
49	6	Assembly (3-dimensional paper form board: mechanical assemblies)	20
50	7	Judgment and Comprehension (Paragraphs with extrapolation and inference questions)	24
51	8	Components (Like Gottschaldt: hidden figures, mostly 3-dimensional)	40
52	9	Planning (Organizational rearrangement: main-step and substep position scored)	32

Table 1 (continued)

Var. No.	Code or Subtest	Title of Test	Max. Score
		Flanagan Aptitude Classification Tests	
53	10	Arithmetic (Add, subtract, mixed add and subtract, count X's, multiply, divide, mixed multiply and divide)	120
54	11	Ingenuity (Like 32: TALENT Creativity)	24
55	12	Scales (Graph reading)	72
56	13	Expression (Grammatical sentences TF--40; best and worst sentence out of 3--12)	64
57	15	Alertness (Find dangerous item in picture)	36
58	17	Patterns (Copying on graph paper: 18 direct and 12 upside down)	60
59	18	Coding (6 categories and 5 subcategories each. Memorize codes: practice exercises. 3 and 6 choice)	120
60	19	Memory (Code memory: 30-choice)	30
		Differential Aptitude Tests	
61	1	Verbal Reasoning (Verbal analogies: 2-blank)	50
62	2	Numerical Ability (Arithmetic computation: easy to hard)	40
63	3	Abstract Reasoning (Figure classification)	50
64	4	Space Relations (Pattern and fold-ups, mult. mark)	100
65	5	Mechanical Reasoning (Bennett)	68
66	6	Clerical Speed and Accuracy (Match pairs of letters and numbers from booklet to answer sheet)	100
67	7-I	Spelling (single words: TF)	100
68	7-II	Sentences (each divided into 5 parts: mark all parts which contain errors in grammar, punctuation, or spelling. 50 items, 250 parts, 95 actual errors. R-W)	95
		General Aptitude Test Battery	
69	1	Name Comparison (Like Minnesota)	150
70	2	Computation (Arithmetic computation easy to hard)	50
71	3	Three-dimensional Space (Pattern and fold-ups)	40
72	4	Vocabulary (same-opposite)	60
73	5	Tool Matching (Identical forms: much more speed than in TALENT and FACT inspection)	49
74	6	Arithmetic Reasoning (Verbal problems)	25
75	7	Form Matching (Two half-pages of same forms in random arrangements)	60
76	8	Mark Making (Make $\overline{"}$ in as many as possible 5/16" square boxes)	200

Table 1 (continued)

Var. No.	Code or Subtest	Title of Test	Max. Score
		General Aptitude Test Battery	
77	9	Peg Board: Place (Move 48 pegs from one part of peg board to the other. 3 trials	144
78	10	Peg Board: Turn (Turn each peg over and replace in same hole. 3 trials)	144
79	11	Rivet Assemble (Pick up rivet, insert washer, and put in corresponding hole on other side of board)	50
80	12	Rivet Disassemble (Remove rivet and washer from hole, put rivet in corresponding hole, put washer on rod)	50
		Essential High School Content Battery	
81	1	Mathematics (Arithmetic, algebra, geometry, graph reading, table reading)	66
82	2	Science (information, reasoning from data)	70
83	3	Social Studies (information, map locations	90
84	4A	Reading (Story and questions)	15
85	4B	Vocabulary	15
86	4C	Business Definitions (3-5 matching)	12
87	4D	Use of References (12-15 matching)	12
88	4E	Literature Acquaintance (information)	15
89	4F	Language usage (Sentences: find errors: TF)	60
90	4G	Capitalization and Punctuation (TF)	60
91	4H	Spelling (Words in sentences: TF)	60

Chapter Three

PROCEDURES AND RESULTS

Each of the two 91-variable correlation matrices was factored to 16 factors. Table 2 gives the means and standard deviations for the 257 boys and the 286 girls.

Table 3 gives data for deciding on the number of salient factors. The scree test, based on the eigenvalue differences, suggests either 8 or 10 factors for the boys, and 8 factors for the girls. For both groups, the normal deviate from the Bargmann test suggests nine factors (see figures in parentheses in Table 3).

Tables 4 and 5 show the initial principal-axes factor matrices to twelve factors. In these and all later tables, decimal points properly preceding each factor loading are omitted. In each table, the largest entry in each of columns 9, 10, 11, and 12 is in parentheses. It is clear that no test will lose any considerable part of its total common variance if we stop at ten factors. This statement becomes somewhat less clear if we stop at eight. And contrary to the results of Table 3, the highest loading for any factor beyond the eighth is on factor 10 for the girls.

In view of these somewhat equivocal and inconsistent results, it was decided to rotate ten factors first for both sets of data. Ten-factor computed communalities from the initial factor matrices were put in the diagonals of the correlation matrices, which were then re-factored. The results agreed essentially with those of Tables 3, 4, and 5. The scree test for the boys showed reversals at eight and again at ten factors, the scree test for the girls showed one reversal at eight, the Bargmann test indicated nine significant factors for each matrix, and the highest loadings on both factors 9 and 10 were for the girls.

The promax rotations for the two samples yielded the following results for rotated factors 9 and 10:

Factor 9: Boys

Var. No.	Code or Subtest	Test Name	Factor Loading
52	FACT-9	Planning	.367
50	FACT-7	Judgment and Comprehension	.302
60	FACT-19	Memory (for code)	.298
59	FACT-18	Coding	.268
54	FACT-11	Ingneuity	.261
44	FACT-1	Inspection	.260
46	FACT-3	Tables	.240
55	FACT-12	Scales	.229
90	EHSCB-4G	Capitalization and Punctuation	.223

Factor 9: Girls

Var. No.	Code or Subtest	Test Name	Factor Loading
43	F440	Object Inspection	.496
41	F420	Table Reading	.412

Factor 10: Boys

Var. No.	Code or Subtest	Test Name	Factor Loading
88	EHSCB-4E	Literature Acquaintance	.308
48	FACT-5	Vocabulary	.289
2	R103	Literature (information)	.281
39	R333	Advanced Mathematics	.280
3	R104	Music (information)	.277

Factor 10: Girls

Var. No.	Code or Subtest	Test Name	Factor Loading
59	FACT-18	Coding	.426
60	FACT-19	Memory (for code)	.324
46	FACT-3	Tables	.272
55	FACT-12	Scales	.244
53	FACT-10	Arithmetic	.228
76	GATB-8	Mark Making	.202

Table 2. Means and Standard Deviations

	Boys (257)		Girls (286)	
Var.	Mean	Std. Dev.	Mean	Std. Dev.
1	19.81	5.31	18.54	5.29
2	13.78	4.44	13.52	4.13
3	6.24	2.89	7.13	2.78
4	15.82	5.04	13.09	4.66
5	10.85	5.52	7.73	4.84
6	10.34	4.04	6.81	3.58
7	7.46	2.24	6.12	2.18
8	6.38	1.75	6.65	1.70
9	5.23	2.34	2.95	1.76
10	9.45	4.27	4.95	2.27
11	13.17	2.96	8.28	2.58
12	9.03	1.77	8.31	2.09
13	9.03	2.85	13.55	3.03
14	8.95	2.85	6.00	2.68
15	6.48	2.29	6.68	2.42
16	5.59	1.71	4.87	1.63
17	6.63	1.66	6.92	1.61
18	4.69	1.92	4.85	2.00
19	8.65	3.54	8.98	3.03
20	4.84	1.97	2.34	1.29
21	5.53	1.88	4.45	1.70
22	8.89	2.92	10.12	2.89
23	11.86	5.33	13.97	5.79
24	14.66	6.05	16.52	6.64
25	9.51	2.76	10.85	2.50
26	30.07	2.49	30.93	2.35
27	18.48	4.35	20.56	3.78
28	17.04	3.14	18.00	2.82
29	8.90	2.23	9.43	1.79
30	12.18	5.83	14.14	5.99
31	33.30	10.33	34.10	8.70
32	10.12	4.12	9.19	3.26
33	13.42	3.52	9.13	3.21
34	14.54	5.60	12.52	5.15
35	9.86	3.08	8.74	2.66
36	9.38	2.79	9.30	2.71
37	9.35	3.48	8.53	3.28
38	11.96	4.93	10.83	4.57
39	4.17	2.61	3.13	1.89
40	28.86	18.15	32.08	16.86
41	11.38	8.25	11.65	4.70
42	25.56	18.15	30.56	16.40
43	21.71	7.57	22.63	6.30
44	50.04	9.42	51.85	9.18
45	13.92	4.80	9.46	2.68
46	49.62	11.79	53.74	12.02
47	10.80	5.10	9.20	4.61
48	21.12	11.04	21.76	10.55
49	11.61	4.07	10.58	3.56

Table 2 (continued)

	Boys (257)		Girls (286)	
Var.	Mean	Std. Dev.	Mean	Std. Dev.
50	15.46	4.02	14.55	3.66
51	23.19	7.79	21.57	7.01
52	22.68	5.30	23.85	4.55
53	48.50	10.67	48.26	10.26
54	15.04	4.52	14.15	4.29
55	26.57	8.42	23.46	7.85
56	42.84	7.80	46.67	7.65
57	26.84	4.68	24.11	4.51
58	17.76	10.79	14.26	9.47
59	106.24	16.60	110.28	14.81
60	18.80	7.55	19.34	7.32
61	26.78	10.08	26.76	10.01
62	23.81	10.16	19.75	10.28
63	30.12	10.22	29.54	10.37
64	52.87	22.39	48.53	19.46
65	44.25	11.44	28.84	10.23
66	58.39	11.80	64.40	9.90
67	55.50	25.45	67.94	21.36
68	39.13	17.59	46.85	16.46
69	53.84	10.36	61.83	11.70
70	26.51	4.73	26.60	4.94
71	20.07	5.27	18.47	4.64
72	21.79	7.49	23.00	7.55
73	33.99	5.05	35.95	4.88
74	13.16	3.19	12.23	3.28
75	30.04	5.69	30.46	5.66
76	69.92	8.95	74.05	7.32
77	89.04	7.81	87.13	7.80
78	97.57	8.21	101.22	8.41
79	27.06	4.07	29.10	4.15
80	28.54	3.13	29.68	3.23
81	34.47	13.32	27.42	11.89
82	41.53	12.06	36.09	10.62
83	42.62	12.99	37.88	11.02
84	10.97	2.26	10.55	2.32
85	10.12	3.23	10.28	3.10
86	7.79	2.12	7.56	2.16
87	7.21	2.50	7.42	2.30
88	7.77	2.70	8.68	2.71
89	42.42	5.96	43.72	6.01
90	48.78	6.91	51.49	5.08
91	46.95	7.73	50.48	6.03

Table 3. Data for Decisions on Numbers of Factors

	Boys				Girls			
Factor	Eigen-value	Diff.*	Per Cent of Trace	Normal Deviate	Eigen-value	Diff.	Per Cent of Trace	Normal Deviate
1	33.28		58	44.87	31.55		58	42.98
		27.41				26.16		
2	5.87		68	33.54	5.40		68	30.02
		1.56				1.88		
3	4.31		76	22.97	3.52		74	20.80
		1.77				1.29		
4	2.54		80	15.90	2.23		78	13.93
		.89				.66		
5	1.65		83	12.48	1.57		81	9.22
		.28				.29		
6	1.37		85	9.04	1.28		84	6.80
		.15				.14		
7	1.22		88	6.12	1.14		86	3.99
		.07				.06		
8	1.15		90	2.68	1.08		88	1.54
		(.17)				(.16)		
9	.98		91	(.24)	.91		89	(-.57)
		.06				.04		
10	.91		93	-2.14	.87		91	-2.58
		(.15)				.05		
11	.76		94	-3.89	.81		93	-4.51
		.03				.06		
12	.73		96	-5.90	.75		94	-6.44
		.03				.07		
13	.70		97	-7.54	.68		95	-7.99
		.09				.05		
14	.61		98	-9.39	.63		96	-9.67
		.02				.01		
15	.59		99	-10.98	.63		97	-11.30
		.03				.06		
16	.56		100	-12.62	.57		99	-12.97

*Eigenvalues and eigenvalue differences were rounded separately. In consequence the reported differences will sometimes differ from the differences between rounded eigenvalues by $\pm$ .01.

Table 4. Initial Factor Matrix: Boys

Var.	I	2	3	4	5	6	7	8	9	10	11	12
1	854	-216	078	169	014	-030	038	-019	-016	-079	-067	020
2	703	-235	-009	250	-112	-168	-036	241	-075	012	061	053
3	622	-126	023	205	-130	-278	041	193	-131	037	-049	-024
4	748	-249	-046	149	-004	154	049	215	-025	-011	041	-079
5	840	-069	-017	-217	102	-025	-105	126	-114	089	-016	-076
6	825	-106	086	064	142	-024	-009	-034	-075	-031	028	-048
7	699	-100	120	194	020	026	-021	-065	-014	014	084	-068
8	631	-169	043	-045	-059	150	-017	-009	078	-132	116	050
9	636	-120	213	176	132	-040	-073	001	029	002	118	-064
10	605	-150	430	116	235	015	-118	-092	-117	022	-060	-158
11	528	-160	260	224	302	135	-108	-150	-024	006	-111	011
12	491	-200	076	260	169	191	-064	-141	-010	115	078	053
13	442	-156	063	203	-022	065	032	004	-133	004	-159	-065
14	517	-092	-215	054	-117	136	-031	183	-025	-042	043	-123
15	664	-146	137	248	-158	024	-000	006	-096	-020	-145	111
16	649	-247	024	218	-100	179	-052	084	004	-079	-012	-015
17	617	-219	044	134	-057	095	009	-090	006	-085	-087	-098
18	558	-228	061	156	-044	165	061	034	003	-236	-172	143
19	687	-208	-026	209	-083	010	090	170	-029	-016	-036	055
20	183	-192	261	236	040	-002	-153	-223	022	074	174	-003
21	579	-140	148	254	028	140	-048	-041	022	-075	043	058
22	296	053	063	-060	-044	129	426	-038	-286	149	117	-060
23	524	-030	-158	-043	-038	-055	363	-076	-220	198	-019	074
24	609	-040	-235	220	-169	-264	-051	-085	-058	-106	-017	146
25	534	-079	-464	-114	-028	-096	-019	-266	-036	036	-164	000
26	629	-028	-230	-106	041	-013	168	029	001	-241	044	-112
27	779	-057	-093	-169	009	-065	150	005	-047	-126	042	-102
28	690	-081	-098	-047	087	-090	061	-100	053	-102	-019	-012
29	593	-018	-120	017	007	017	173	014	039	-087	122	-070
30	739	-007	-097	-270	-044	-135	065	-005	-049	-018	-007	-050
31	836	-214	013	-012	-065	054	143	036	117	-052	087	027
32	718	-039	235	063	046	018	089	-071	-033	005	-080	030
33	584	153	540	-014	019	-089	057	-171	-060	-088	085	-021
34	353	276	275	029	-116	-057	049	124	-011	-099	016	039
35	504	339	413	-164	-203	-072	-014	-025	-069	003	-065	-022
36	633	199	151	-247	-219	-057	049	005	007	(-289)	030	-030
37	739	-080	027	-198	088	057	018	017	-015	-178	-065	096
38	788	012	-084	-337	077	-036	-047	086	-110	001	017	007
39	606	037	044	-283	171	-189	-178	162	-117	212	094	008
40	533	109	-335	-077	114	233	-092	032	-177	-115	-041	-043
41	115	315	-196	261	-149	020	-121	-170	-218	-120	(292)	065
42	236	310	-289	260	-223	-116	-121	-145	-112	-165	213	052
43	115	428	-016	175	-159	-197	027	-025	-247	-035	045	004
44	328	617	145	197	-113	018	-037	039	173	098	010	-100
45	581	-049	358	-036	187	-019	-117	-054	-141	074	054	-147
46	488	535	-308	102	035	145	-048	-082	012	103	064	030
47	813	064	-001	-248	053	-023	-118	047	018	078	011	103
48	754	-254	-042	117	-092	-278	-120	080	-006	054	-071	123
49	501	384	198	-193	-177	005	075	104	084	175	144	088
50	720	-115	174	006	036	138	-018	-013	248	031	040	078

Table 4 (continued)

Var.	1	2	3	4	5	6	7	8	9	10	11	12
51	510	340	200	065	-082	096	-064	-020	-175	129	-131	059
52	653	-031	-027	035	008	123	051	026	(318)	021	-006	080
53	497	388	-470	-048	086	181	-166	089	-080	-026	155	090
54	704	005	061	092	-031	004	-026	-098	206	123	083	168
55	566	499	-018	010	-021	150	-041	-123	065	032	-003	025
56	793	-075	-114	-157	009	-186	105	-103	117	029	004	-030
57	148	396	172	410	-084	024	073	054	015	038	-007	-029
58	436	408	155	-231	-027	090	-003	-155	-036	-050	-104	(266)
59	346	157	-230	143	-013	265	183	-091	-002	090	-046	-008
60	466	170	-171	051	-054	252	335	-061	-072	225	-167	103
61	869	-162	053	-060	-108	-038	049	-043	063	-007	-002	081
62	811	077	-120	-258	-003	091	-107	028	-000	-003	045	039
63	700	172	198	-253	-151	005	080	042	139	-081	017	099
64	593	377	416	-234	-112	-018	050	-040	137	-032	-007	013
65	528	111	550	-109	056	-005	-019	-211	-018	-045	020	-052
66	220	613	-298	214	-032	-011	056	039	018	029	082	-111
67	620	-174	-432	-062	037	-117	-067	-318	031	029	-083	-008
68	798	-121	-116	-105	-027	-174	068	-086	037	031	-093	-007
69	460	383	-403	101	-090	-009	-153	-120	080	-002	-182	-167
70	624	218	-388	-206	113	182	-169	087	-052	-053	-033	-003
71	446	424	461	027	-074	-032	-091	-003	-098	-001	-074	-049
72	840	-181	-085	171	056	-134	-103	-024	075	-037	-012	071
73	274	561	-057	195	-129	-058	-072	117	163	-079	-219	-222
74	784	090	-184	-145	054	134	-234	063	-038	-078	-027	057
75	332	565	087	059	-101	-053	-144	017	102	119	-108	-082
76	157	278	-406	219	311	-158	166	021	071	-107	023	060
77	128	435	-039	170	446	-104	157	027	139	032	001	082
78	233	312	014	131	502	-266	129	121	076	-144	-043	052
79	192	401	182	019	274	030	107	129	-135	-132	-007	156
80	283	499	041	114	311	-094	071	119	-040	029	-045	-060
81	806	075	-085	-223	097	042	-176	108	-095	120	-004	067
82	880	-142	080	022	068	001	-011	-024	-009	104	071	001
83	748	-261	-055	074	-056	079	-047	227	-024	065	-044	-030
84	699	-100	059	011	069	072	-002	033	097	046	216	-110
85	779	-202	-008	043	-108	-135	029	043	047	047	-024	015
86	585	-192	-040	042	029	132	-002	161	-006	025	-066	-106
87	686	-023	-014	-043	-152	-016	-021	089	082	128	-032	-108
88	177	-102	-201	243	-078	-124	-056	142	070	218	054	215
89	739	-125	-090	-091	028	-173	103	-016	098	103	089	-120
90	647	004	-160	-040	-063	103	185	-051	112	-009	133	-198
91	671	-174	-354	-024	017	-131	-045	-288	002	110	-037	-102

Table 5. Initial Factor Matrix: Girls

Var.	1	2	3	4	5	6	7	8	9	10	11	12
1	831	-317	-081	102	-040	-057	-056	-009	-028	-009	-030	037
2	730	-347	-024	145	-080	106	-084	-028	091	045	111	-056
3	671	-152	-146	119	027	043	-103	042	-091	045	055	-054
4	674	-307	024	199	-120	073	028	-039	002	058	110	114
5	773	-028	145	-279	-189	-010	-140	-002	100	-005	029	053
6	667	-296	137	-063	-046	058	-081	-032	140	-068	-049	077
7	635	-334	-029	060	-035	049	002	-052	030	-078	015	173
8	557	-111	000	012	037	-013	123	-002	-046	-080	014	-150
9	425	-276	129	155	-113	090	-117	-065	090	210	077	067
10	341	-170	160	088	-194	-022	025	002	076	-077	(-277)	147
11	404	-196	201	176	085	-223	142	005	157	061	-225	145
12	605	-222	064	206	-053	-195	189	100	020	-004	-086	019
13	442	-033	069	113	097	-377	145	018	005	-015	050	150
14	608	-094	055	159	-155	175	083	-032	-023	027	-053	031
15	630	-308	-028	274	014	016	-151	012	018	-086	078	-013
16	536	-263	009	106	-164	-046	100	-038	047	031	137	-060
17	565	-193	-095	030	-113	-201	039	-094	005	024	059	029
18	619	-276	053	092	-120	-039	029	-021	-099	-007	-044	-036
19	611	-322	-061	062	-047	060	082	-083	-064	014	055	-124
20	053	-124	-018	135	015	028	093	072	198	155	028	196
21	522	-168	107	092	-027	-090	005	-004	211	-010	090	-067
22	408	080	-046	027	098	408	115	-043	259	-097	-095	-070
23	496	-061	-041	-045	262	323	-071	-115	214	080	-050	-042
24	672	058	-283	056	159	-022	-234	016	-034	123	-086	079
25	474	074	-465	-076	204	003	-051	-006	199	-050	-078	115
26	550	193	-217	170	195	036	052	023	099	-264	130	027
27	735	087	-141	-185	118	-174	-021	022	009	013	117	-068
28	613	-057	-214	-044	143	-011	002	081	102	-129	052	-103
29	538	-161	-123	036	179	049	-083	001	048	-101	-214	-110
30	770	086	-021	-149	083	014	-081	-108	-092	004	011	-022
31	830	-182	-014	074	053	064	034	-038	-067	015	-012	-015
32	631	-070	109	137	072	234	-119	-088	-016	-059	-092	-055
33	547	020	346	089	253	-017	-029	-065	114	010	-032	149
34	391	249	260	-084	136	-052	-075	067	051	131	014	011
35	435	142	508	-026	195	060	-054	-003	-047	-007	041	107
36	639	154	233	-044	157	043	-005	-095	-067	-098	141	020
37	705	-019	117	-258	-094	-028	118	-074	-109	012	-027	019
38	742	078	089	-382	-110	040	-013	-107	045	-135	-039	028
39	446	094	169	-308	-148	186	-265	009	090	-108	098	075
40	568	334	-222	-096	027	-007	253	086	014	-162	042	116
41	204	443	-147	149	192	200	016	-076	-189	194	007	210
42	318	210	-207	067	-042	001	-065	042	-185	111	-131	(228)
43	175	334	108	191	173	128	-170	-080	(-315)	053	130	178
44	341	433	099	319	-123	022	-094	163	-093	056	-038	-051
45	274	000	133	044	-087	-005	-085	-141	074	-135	-233	158
46	484	510	-126	133	-170	094	110	107	019	123	-108	-054
47	726	-034	157	-327	-182	021	-073	-097	-029	-019	036	-048
48	765	-295	-089	024	-104	029	-238	055	022	099	026	026
49	481	165	406	024	017	-129	063	091	-019	-038	-029	-004
50	749	-143	115	010	015	-003	081	-059	-224	-042	-073	-096

Table 5 (continued)

Var.	1	2	3	4	5	6	7	8	9	10	11	12
51	460	233	286	054	105	-023	-182	122	139	191	112	-106
52	605	-003	-043	032	115	058	229	014	-010	-086	-078	026
53	578	492	-201	-171	-254	046	111	072	007	066	-115	074
54	655	-119	056	117	-002	-061	021	003	-055	008	-094	-149
55	563	398	189	048	-051	-058	113	107	144	139	-091	-046
56	784	-011	-226	-055	164	-071	-082	064	-158	071	-088	-154
57	320	065	214	471	-095	110	106	067	-045	-151	-043	-054
58	465	271	343	-056	094	-052	056	138	073	132	083	-169
59	361	121	-073	036	-109	008	246	-132	076	(338)	178	004
60	528	027	049	-029	053	346	206	-007	068	202	033	-003
61	892	-149	036	-088	033	-104	004	-040	-075	074	-024	-060
62	766	137	125	-372	-071	-003	079	-012	-054	010	001	005
63	702	133	175	-114	217	059	056	-057	-100	-055	005	043
64	584	199	514	-056	196	-052	-011	099	-061	049	-050	-088
65	619	-004	329	058	182	-036	063	-078	-019	066	-078	-107
66	316	434	-172	216	-201	256	018	114	-007	-063	-065	-103
67	569	017	-480	-103	137	-173	-004	110	109	150	-155	-021
68	788	-011	-254	-091	102	-066	-050	063	-041	064	-134	-177
69	572	373	-293	115	-206	038	-117	184	-026	-049	036	-038
70	695	376	-123	-224	-176	-007	095	105	-022	013	012	103
71	523	288	383	123	064	-057	-052	215	122	-090	102	046
72	856	-142	-203	053	-070	-052	-071	056	-032	081	-004	-005
73	451	353	-058	187	-169	015	-045	203	-011	-238	149	011
74	747	259	063	-296	-144	001	102	-014	-049	002	-022	062
75	410	428	067	219	-054	-116	-186	176	077	-062	-036	-000
76	229	448	-345	139	-072	-033	051	-307	097	048	096	-072
77	085	468	-020	098	-179	-111	-126	-324	070	005	-124	-127
78	200	438	-050	288	-043	-143	016	-427	061	-020	077	019
79	310	434	074	150	053	-173	-058	-253	056	-095	021	047
80	219	453	062	074	-058	-163	-111	-295	016	-044	-070	-134
81	794	076	143	-303	-224	021	-043	-010	047	-022	042	048
82	827	-269	011	009	-061	-042	017	-002	021	-011	-037	037
83	779	-317	-022	082	-165	042	064	-042	-046	060	094	-049
84	678	-169	-008	071	-011	081	159	-073	-147	-048	-064	-029
85	762	-103	-117	044	068	-009	-072	086	-153	-038	-014	058
86	556	-174	057	098	-073	-059	090	021	-062	-042	136	027
87	700	-053	-068	018	-003	-002	-044	-017	-147	-023	017	076
88	315	-169	-091	063	-152	-133	-228	062	067	043	115	013
89	733	-047	-210	-108	137	-013	-056	-054	-011	-063	010	-035
90	600	127	-258	-033	212	-036	242	019	046	-140	245	004
91	663	024	-486	-066	183	-115	-069	023	064	024	051	087

These tables include, for each factor, all positive loadings of .200 or higher. The highest negative loading for any one of them was -.206 (Var. 36: R290, Abstract Reasoning) on Factor 10: Boys. For the other three factors, the highest negative loadings were all smaller than -.200.

For Factor 9: Boys, there is no clear substantive interpretation. About the only thing the tests have in common is that all but the last come from the FACT battery. The best interpretation would seem to be that this is a time-associated error factor.

Factor 9: Girls is a doublet of not-too-clear meaning. In the non-TALENT batteries there are other tests quite similar to Object Inspection and Table Reading. This would seem to be a small perceptual-speed factor, emerging as a separate doublet only because both of these tests were administered consecutively at the same test session.

Factor 10: Boys includes the two literary knowledge tests, but only one of the four vocabulary tests, and it is not clear why Advanced Mathematics is related to these tests. With highest loading .308, it is probably best interpreted as an error factor.

Factor 10: Girls is fairly similar to Factor 9: Boys. It seems to be mainly a FACT factor, dominated by a doublet generated by lack of experimental independence of the FACT Coding and Memory tests.

Since none of these factors permitted any clear substantive interpretation, new communalities were computed for the first eight factors of Tables 4 and 5 and put into the diagonals of the correlation matrices, which were then re-factored. The results were again fairly similar to those of Table 3: the scree test showed reversals at eight and ten factors for the boys and at eight factors only for the girls, and the Bargmann test again suggested nine statistically significant factors. In this case, however, the highest loadings on both factors 9 and 10 were for the boys. The eight-factor principal axes matrices are shown in Tables 6 and 7.

Since this was the main study, each promax rotation was followed by eight single-hyperplane rotations. Complete consistency was not

Table 6. Principal - Axes Factor Matrix: Boys

Var.	I	2	3	4	5	6	7	8	Communalities
1	855	-218	080	174	013	-033	038	019	818
2	704	-237	-008	255	-117	-156	-081	-259	728
3	621	-126	024	206	-130	-274	-023	-203	579
4	748	-249	-045	151	-008	157	056	-209	717
5	842	-069	-017	-223	110	-010	-128	-131	809
6	825	-106	087	066	146	-018	-013	028	726
7	697	-099	119	189	021	031	-011	060	551
8	629	-166	042	-045	-055	148	020	005	453
9	635	-119	211	174	129	-026	-071	003	515
10	605	-149	429	116	240	029	-124	099	670
11	527	-159	260	223	304	146	-087	163	569
12	490	-198	075	252	160	190	-036	140	432
13	440	-153	062	191	-023	057	019	-005	261
14	515	-089	-206	051	-108	130	-018	-153	371
15	663	-145	136	243	-156	026	-015	-005	563
16	648	-246	025	215	-101	184	-038	-075	579
17	615	-217	044	130	-054	088	026	092	464
18	555	-223	061	148	-043	143	073	-031	412
19	687	-208	-024	209	-086	009	075	-176	603
20	182	-185	249	213	038	011	-125	185	225
21	578	-138	146	245	025	138	-024	041	456
22	294	052	061	-053	-041	076	341	-005	219
23	522	-030	-153	-038	-039	-082	297	035	395
24	608	-041	-233	219	-169	-251	-082	073	577
25	533	-079	-463	-112	-030	-101	-021	271	603
26	627	-027	-225	-098	038	-024	171	-040	486
27	779	-057	-093	-167	009	-077	152	-022	676
28	688	-080	-097	-043	084	-094	065	085	519
29	592	-018	-118	019	003	001	182	-029	398
30	739	-007	-097	-269	-044	-141	054	-004	653
31	837	-215	015	-010	-069	048	171	-039	785
32	717	-038	231	064	044	009	083	061	585
33	585	154	541	-014	023	-097	056	158	697
34	352	271	267	029	-108	-051	038	-116	299
35	504	342	414	-166	-205	-076	-030	026	620
36	632	199	148	-240	-207	-061	057	-020	568
37	738	-079	026	-193	087	055	034	-026	601
38	789	014	-086	-342	083	-028	-060	-101	769
39	605	038	042	-279	173	-160	-218	-157	575
40	531	108	-329	-072	106	227	-069	-029	475
41	115	306	-188	243	-135	026	-110	125	248
42	235	303	-279	248	-209	-098	-122	114	368
43	115	419	-016	167	-148	-181	-020	-005	272
44	327	613	143	196	-114	021	-026	-012	556
45	580	-047	355	-036	188	-005	-131	051	521
46	488	535	-308	106	028	144	-029	090	661
47	814	066	-002	-251	056	-011	-121	-046	749
48	755	-256	-041	119	-095	-270	-170	-084	769
49	499	380	194	-186	-171	-001	068	-096	509
50	718	-113	172	006	032	137	023	032	580

Table 6 (continued)

Var.	1	2	3	4	5	6	7	8	Communalities
51	509	337	195	064	-077	093	-076	022	435
52	651	-030	-025	034	004	109	090	-000	447
53	497	391	-475	-045	086	204	-154	-092	708
54	702	005	059	088	-032	007	-006	104	516
55	565	499	-019	014	-024	145	-012	130	608
56	793	-075	-114	-155	009	-199	108	102	733
57	148	392	169	402	-088	016	059	-045	379
58	434	400	147	-215	-024	074	016	132	440
59	345	155	-224	140	-017	222	190	076	304
60	464	167	-167	053	-057	197	313	045	416
61	870	-164	054	-060	-112	-044	057	040	809
62	812	078	-122	-261	-002	104	-092	-024	768
63	700	174	198	-252	-152	-004	102	-042	659
64	593	382	418	-237	-115	-028	070	045	749
65	528	113	553	-112	062	-008	-007	217	661
66	220	611	-297	217	-038	-015	052	-043	563
67	620	-176	-436	-061	036	-122	-061	338	744
68	798	-121	-116	-104	-027	-183	059	084	720
69	460	380	-400	103	-092	002	-136	139	573
70	624	220	-392	-206	115	205	-152	-080	719
71	445	425	459	026	-071	-023	-109	005	608
72	841	-183	-086	178	056	-126	-110	031	812
73	273	548	-056	186	-122	-047	-063	-082	440
74	785	091	-187	-146	056	163	-219	-055	762
75	331	558	083	059	-098	-038	-138	007	462
76	157	277	-405	226	303	-170	164	-038	465
77	128	432	-040	172	425	-115	154	-035	452
78	233	313	013	139	505	-279	118	-137	537
79	191	394	175	022	252	020	091	-130	312
80	283	494	039	117	294	-094	050	-111	449
81	806	077	-087	-226	102	064	-193	-108	778
82	881	-143	081	023	070	007	-013	025	809
83	748	-216	-054	075	-057	091	-053	-210	694
84	698	-098	059	012	063	075	020	-025	510
85	779	-201	-007	043	-108	-133	013	-041	680
86	584	-189	-039	042	024	128	003	-136	415
87	684	-023	-012	-041	-146	-011	-022	-069	497
88	175	-099	-189	221	-068	-091	-067	-104	153
89	739	-125	-089	-089	028	-179	-097	011	620
90	645	004	-156	-037	-064	080	206	043	496
91	671	-175	-355	-024	017	-134	-049	304	721

Table 7. Principal - Axes Factor Matrix: Girls

Var.	I	2	3	4	5	6	7	8	Communalities
1	833	-321	-084	103	044	056	-055	025	823
2	730	-348	-026	146	089	-109	-054	075	705
3	670	-151	-147	118	-021	-058	-100	000	521
4	674	-306	021	195	130	-060	050	028	610
5	774	-029	150	-280	183	009	-138	071	758
6	666	-293	136	-057	046	-058	-050	069	563
7	633	-331	-030	058	040	-040	032	051	522
8	555	-109	000	014	-034	021	103	-044	334
9	424	-270	123	146	114	-086	-061	103	324
10	340	-165	151	080	178	028	019	-018	205
11	402	-192	192	173	-081	215	086	-064	330
12	604	-220	060	203	053	200	102	-171	541
13	441	-033	066	115	-101	363	066	-085	367
14	607	-093	052	153	161	-153	110	-005	464
15	630	-307	-033	273	-005	-036	-132	054	588
16	534	-255	007	097	151	057	089	003	393
17	564	-190	-094	027	103	208	044	056	422
18	617	-272	050	088	117	048	030	-002	482
19	609	-318	-060	058	047	-041	113	037	497
20	052	-119	-019	118	-007	-022	047	-068	038
21	520	-164	102	089	027	084	-012	012	324
22	406	078	-043	026	-076	-353	150	030	327
23	496	-062	-039	-039	-250	-323	018	159	446
24	671	058	-282	056	-157	-013	-218	068	613
25	473	073	-453	-075	-199	-016	-042	043	483
26	549	189	-213	164	-182	-042	048	-023	447
27	734	087	-138	-179	-132	164	-052	-012	645
28	611	-056	-209	-042	-139	-000	-030	-060	446
29	535	-157	-116	036	-158	-054	-052	031	357
30	769	085	-018	-142	-090	-017	-022	128	645
31	830	-184	-015	077	-051	-062	060	016	739
32	630	-070	106	140	-056	-235	-038	133	510
33	546	019	339	099	-241	008	-003	071	487
34	390	244	257	-072	-135	029	-099	-028	313
35	435	140	505	-010	-190	-071	-035	026	506
36	638	151	231	-031	-152	-038	037	087	517
37	705	-019	121	-254	084	051	136	009	605
38	742	078	096	-384	101	-025	041	115	740
39	466	094	173	-304	142	-205	-224	122	476
40	567	331	-216	-096	-033	019	195	-166	554
41	204	433	-144	142	-171	-187	071	047	342
42	316	201	-195	056	036	-010	-060	-027	187
43	175	325	103	185	-153	-135	-086	110	242
44	341	432	095	316	127	-045	-142	-120	464
45	273	001	125	041	080	014	-017	134	117
46	484	510	-128	128	171	-091	072	-143	590
47	727	-035	164	-332	178	-010	-031	123	714
48	765	-296	-091	023	109	-054	-244	051	759
49	480	162	400	034	-022	121	007	-106	444
50	748	-143	113	014	-013	018	098	-003	602

Table 7 (continued)

Var.	1	2	3	4	5	6	7	8	Communalities
51	459	228	277	062	-101	-011	-208	-024	397
52	604	-003	-042	033	-108	-039	205	-101	433
53	579	498	-202	-182	254	034	080	-166	743
54	653	-117	053	113	003	058	006	-026	460
55	561	393	186	052	045	058	044	-130	531
56	784	-011	-225	-052	-171	049	-104	-045	712
57	318	063	201	447	105	-095	089	-101	384
58	464	267	337	-042	-096	039	-014	-132	430
59	359	117	-070	031	095	027	220	020	207
60	527	025	048	-027	-041	-304	212	-053	423
61	893	-151	038	-084	-041	112	007	022	844
62	768	139	133	-379	060	017	081	-026	782
63	701	132	176	-101	-216	-055	083	023	608
64	584	198	517	-039	-203	035	-053	-095	703
65	618	-005	324	068	-175	042	084	040	532
66	316	431	-173	203	204	-252	011	-095	470
67	568	017	-473	-103	-147	150	-071	-101	617
68	787	-001	-252	-089	-111	051	-075	-049	714
69	573	376	-297	109	210	-064	-186	-123	667
70	695	379	-122	-231	169	012	043	-143	746
71	522	286	380	132	-063	030	-145	-159	567
72	857	-144	-208	051	072	044	-100	-031	819
73	449	347	-058	176	162	-032	-106	-142	415
74	748	262	068	-302	137	017	102	-034	755
75	410	427	064	221	053	079	-258	-085	486
76	228	447	-346	134	070	064	155	262	491
77	085	467	-021	096	172	132	-005	335	394
78	201	443	-055	297	046	191	174	401	557
79	309	429	071	151	-055	176	024	236	398
80	219	448	061	075	048	169	-013	283	369
81	795	077	150	-311	222	-013	-036	035	809
82	828	-270	010	011	063	049	010	-010	765
83	779	-318	-024	080	170	-024	085	013	752
84	678	-169	-008	070	014	-056	188	-009	531
85	760	-102	-115	043	-065	-007	-087	-059	618
86	555	-172	055	095	071	066	064	-054	366
87	699	-053	-068	019	000	-003	-023	022	497
88	313	-162	-086	053	128	093	-202	035	201
89	733	-047	-207	-105	-141	006	-026	071	619
90	599	126	-253	-030	-209	045	205	-097	536
91	664	024	-487	-068	-196	096	-090	008	739

reached for all factors, but except in one doubtful case, all negative loadings were reduced to values consistent with positive-manifold interpretations. It should be noted that neither of the original correlation matrices had a negative column sum, and that no row of either Table 6 or Table 7 had initially a negative element in the first column.

Tables 8 and 9 show the rotated factor matrices. In these tables, and in all later tables, factor loadings are entered only if their numerical values are .250 or above. The first column of each table gives the variable number, to facilitate reference back to Table I. The second column gives code symbols for the Project TALENT tests, and battery abbreviations and subtest numbers for the other tests. The third column gives test names abbreviated to not more than five characters. For the Project TALENT tests they are abbreviations of the TALENT titles. For the other tests, I have substituted in many cases abbreviations of similar TALENT tests. Thus the FACT Ingenuity test and the TALENT Creativity test are similar, so the abbreviated title for FACT Ingenuity is CREAT. The other columns show the factor loadings, with somewhat less abbreviated headings naming the factors.

In each table (8 and 9), the factors are in order from left to right as they came. Comparable factors are <u>not</u> in most cases, in the same columns. Tables 10 and 11 give the transformation matrices. They transform Tables 6 and 7 respectively into Tables 8 and 9, with the columns of Tables 8 and 9 in the numbered orders in which they appear.

In both tables (8 and 9), the first factor is a large <u>Verbal and Information</u> factor, with substantial loadings on most of the information tests, the vocabulary tests, and the reading tests. The creativity tests, which are really verbal-ingenuity tests, also have moderate loadings on this factor.

The <u>Space and Reasoning</u> factor (2 for boys; 3 for girls) is of some interest because in both analyses what might have been expected to emerge as two different factors came out as one. Substantial loadings appear on all or most of the visualization (space)

test, the mechanical reasoning tests, the object inspection (identical forms) tests, the FACT Hidden Figures test (similar to the Gottschaldt), the FACT Scales (graph-reading) test, the FACT Patterns (copying designs) test, and the GATB Form Matching test. For the boys, the TALENT and DAT Abstract Reasoning (figure matrices and figure classification) tests have substantial loadings on this factor; for the girls these tests have no loading as high as .250 on any factor.

It is interesting to note that the DAT Verbal Reasoning (2-blank verbal analogies) test has no loading as high as .250 on either the Verbal and Information factor or the Space and Reasoning factor. It is the only verbal reasoning test in any of the batteries.

For the girls there are three loadings above .250 but below .300 on more or less irrelevant tests [Mechanics (information), Home Economics (information), and Rivet Assembly]. For the boys, the only negative loading above -.200 appears: on the GATB Mark Making test.

The one hidden-figures test has nothing to go with it to form a perceptual closure factor. It and the perceptual-speed tests all have some spatial content and in some cases a little reasoning content.

The Clerical and Perceptual factor (3 for boys; 2 for girls) loads mainly on the clerical-speed and perceptual-speed tests. The latter, as noted above, load also on the Space and Reasoning factor. The table-reading tests load also on the Clerical and Perceptual factor, and a number of other tests having substantial speed and accuracy content have moderate loadings. For the boys the clearly clerical tests have the highest loadings; for the girls this effect is less marked. Two spelling tests have loadings above .300 for the boys, while for the girls these tests have near-zero loadings (.006 and -.033).

The Mathematics factor (6 for boys; 4 for girls) has somewhat higher loadings on the computation tests for the boys, and on the arithmetic reasoning and high school mathematics tests for the girls.

Table 8. Rotated Factor Matrix: Boys

Var.	Code	Name	1 Verbal Inf	2 Space Reas	3 Cler Percep	4 Mechan Outdr	5 Coord	6 Math	7 Memory	8 Spell Engl
1	R102 + R162	Vocab	410							
2	R103	Lit	476							
3	R104	Music	317							
4	R105	Soc-S	468							
5	R106	Mat-I						327		
6	R107	Phy-S	318							
7	R108	Bio-S	381							
8	R109	S-Att	253							
9	R110	Aer-S	392							
10	R111	Elec	415			412				
11	R112	Mec-I	443			464				
12	R113	Farm	428			344				
13	R114	Ho-Ec	327							
14	R115	Sport	256							
15	R131	Art	443							
16	R132	Law	507							
17	R133	Hlth	335							
18	R139	Ac + Bu	371							
19	R142	Bible	414							
20	R145 + R146	Hu + Fl	302			340				
21	R147	Outdr	452							
22	R211	Mem-S							344	
23	R212	Mem-W							340	
24	R220	Dis-W			288					375
25	R231	Spell			313					419
26	R232	Cap								
27	R233	Punc								
28	R234	Usage								
29	R235	Effec								
30	R240	Wd-Fu								
31	R250	Read	264							
32	R260	Creat	268							
33	R270	Mec-R		506		295				
34	R281	Vis-2		357						
35	R282	Vis-3		642						
36	R290	Abstr		441						
37	R311	Ar-Rs								
38	R312	Mat-9						311		
39	R333	Mat-A						318		
40	F410	Ar-Co						346		
41	F420	Table			459					
42	F430	Cler			507					251
43	F440	Obj-I		283	303					
44	FACT- 1	Obj-I		431	386					
45	2	Mec-R	262			290				
46	3	Table			554					
47	4	Ar-rs						287		
48	5	Vocab	339							

Table 8 (continued)

Var.	Code	Name	1 Verbal Inf	2 Space Reas	3 Cler Percep	4 Mechan Outdr	5 Coord	6 Math	7 Memory	8 Spell Encl
49	FACT-6	Vis-3		473						
50	7	Read	314							
51	8	Hid-F		363						
52	9	Plan								
53	10	Ar-co			425			449		
54	11	Creat	250							
55	12	Scale		332	423					
56	13	Usage								332
57	15	Alert			269					
58	17	Patrn		430						
59	18	Code			263					
60	19	Mem-C							334	
61	DAT-1	V-Rs								
62	2	Ar-co						327		
63	3	Abstr		421						
64	4	Vis-3		638						
65	5	Mec-R		493		363				
66	6	Cler			495					
67	7-1	Spell			319					477
68	7-11	Engl								310
69	GATB-1	Cler			574					
70	2	Ar-co			263			483		
71	3	Vis-3		574						
72	4	Vocab	374							
73	5	Obj-1		288	388					
74	6	Ar-rs						426		
75	7	Form		428	369					
76	8	Mark		-261	262		441		254	254
77	9	Peg-P					516			
78	10	Peg-T					634			
79	11	Riv-A					314			
80	12	Riv-D					393			
81	EHSCB-1	Math						410		
82	2	Sci	325							
83	3	Soc-S	432							
84	4A	Read	274							
85	4B	Vocab	263							
86	4C	Bus-D	320							
87	4D	Ref								
88	4E	Lit								
89	4F	Usage								254
90	4G	Cap + P								
91	4H	Spell			286					453

Table 9. Rotated Factor Matrix: Girls

Var.	Code	Name	1 Verbal Inf	2 Cler Pecep	3 Space Reas	4 Math	5 Spell Engl	6 Memory	7 Arith Cler	8 Coord
1	R102 + R162	Vocab	400							
2	R103	Lit	436							
3	R104	Music					256			
4	R105	Soc-S	491							
5	R106	Mat-I				416				
6	R107	Phy-S	280							
7	R108	Bio-S	366							
8	R109	S-Att								
9	R110	Aer-S	359							
10	R111	Elec	299							
11	R112	Mec-I	301		259					
12	R113	Farm	406							
13	R114	Ho-Ec			299					
12	R115	Sport	391							
15	R131	Art	394							
16	R132	Law	403							
17	R133	Hlth	316							
18	R139	Ac + Bu	390							
19	R142	Bible	384							
20	R145 + R146	Hu + Fi								
21	R147	Outdr	277							
22	R211	Mem-S						397		
23	R212	Mem-W						410		
24	R220	Dis-W					451			
25	R231	Spell					365			
26	R232	Cap								
27	R233	Punc					270			
28	R234	Usage					268			
29	R235	Effec								
30	R240	Wd-Fu								
31	R250	Read	331							
32	R260	Creat	256					297		
33	R270	Mec-R			368					
34	R281	Vis-2			332					
35	R282	Vis-3			409					
36	R290	Abstr								
37	R311	Ar-Rs				294				
38	R312	Mat-9				442				
39	R333	Mat-A				413				
40	F410	Ar-Co							344	
41	F420	Table						309		
42	F430	Cler								
43	F440	Obj-I								
44	FACT-I	Obj-I		455	329					

Table 9 (continued)

Var.	Code	Name	1 Verbal Inf	2 Cler Pecep	3 Space Reas	4 Math	5 Spell Engl	6 Memory	7 Arith Cler	8 Coord
45	FACT-2	Mec-R								
46	3	Table		469					334	
47	4	Ar-Rs				460				
48	5	Vocab	274				279			
49	6	Vis-3			404					
50	7	Read	303							
51	8	Hid-F			394					
52	9	Plan								
53	10	Ar-Co		372		316			368	
54	11	Creat	287							
55	12	Scale		292	317					
56	13	Usage					375			
57	15	Alert	390	314						
58	17	Patrn			389					
59	18	Code								
60	19	Mem-C						359	320	
61	DAT -1	V-Rs								
62	2	Ar-Co				362				
63	3	Abstr								
64	4	Vis-3			524					
65	5	Mec-R			317					
66	6	Cler		494					303	
67	7-1	Spell					430			
68	7-11	Engl					336			
69	GATB-1	Cler		448			297			
70	2	Ar-Co		294		283			305	
71	3	Vis-3		278	515					
72	4	Vocab	271				283			
73	5	Obj-1		413						
74	6	Ar-Rs				371				
75	7	Form		349	390		265			
76	8	Mark								419
77	9	Peg-P				291				460
78	10	Peg-T								598
79	11	Riv-A			264					376
80	12	Riv-D								406
81	EHSCB-1	Math				457				
82	2	Sci	360							
83	3	Soc-S	482							
84	4A	Read	358							
85	4B	Vocab					272			
86	4C	Bus-D	321							
87	4D	Ref								
88	4E	Lit								
89	4F	Usage					258			
90	4G	Cap+P								
91	4H	Spell					467			

Table 10. Transformation Matrix: Boys

Unrotated Factor	Rotated Factor 1	2	3	4	5	6	7	8
1	.286	.144	.112	.104	.019	.128	.080	.117
2	-.274	.501	.508	-.059	.121	.126	-.003	-.016
3	.224	.550	-.391	.259	.003	-.147	-.078	-.278
4	.680	-.271	.365	.224	.172	-.316	.002	.072
5	-.014	-.489	-.163	.351	.745	.252	.120	-.025
6	.376	-.207	.093	.138	-.360	.512	-.053	-.730
7	-.351	-.094	-.310	-.285	.418	-.526	.979	.103
8	-.258	.247	.558	.803	-.311	-.494	-.109	.599

Table 11. Transformation Matrix: Girls

Unrotated Factor	Rotated Factor 1	2	3	4	5	6	7	8
1	.254	.090	.112	.118	.156	.096	.097	.007
2	-.404	.389	.288	.141	.013	.029	.155	.196
3	.104	-.027	.522	.150	-.432	.050	-.113	-.055
4	.585	.303	.353	-.543	.130	.256	-.078	.173
5	.437	.378	-.440	.648	-.416	-.447	.274	.083
6	.067	-.365	.373	-.003	.142	-.689	-.349	.356
7	.411	-.126	-.306	-.009	-.747	.451	.686	.206
8	.233	-.675	-.284	.478	-.147	.210	-.529	.867

In each table, there is one factor which does not appear in the other. For the boys, it is a Mechanics and Outdoor factor (4). Its main loadings are on the mechanical information and reasoning tests, but it also has substantial loadings on the Farming and Hunting and Fishing tests. It is perhaps as close to a masculinity factor as can be generated by aptitude and information tests, with substantial loadings on the tests on which boys usually make higher scores than girls.

For the girls, the unmatched factor (7) is somewhat weaker, with no loading as high as .400. Its highest loading is on FACT Arithmetic Computation, which is the only number-speed test, with no really harc problems, in the combined batteries. I have termed it an Arithmetic-Clerical factor because it has substantial loadings on most of the arithmetic computation tests, one table-reading test, the one code-memory test which follows and is based on a code-substitution test, and one clerical speed and accuracy test. It is probably as nearly a number- speed test as can be generated by these data.

The Memory factor (7 for boys; 6 for girls) is fairly small. It has substantial loadings for both boys and girls on only the TALENT Memory for sentences and Memory for Words tests and the FACT Memory (for code) test. For the boys it has one other loading of .254, on the GATB Mark Making test. For the girls it has also loadings of .297 on the TALENT Creativity test and of .309 on the TALENT Table Reading test. In each group, the loadings on the named memory tests are all higher than the other loadings.

What should be the English factor is so dominated by the spelling tests for both boys and girls that I have termed it a Spelling and English factor (8 for boys; 5 for girls). For the boys, every loading above .400 is on a spelling test. The other substantial loadings include only a few of the English tests, and also TALENT Disguised Words (recognition of badly misspelled words; e.g., SURKL = circle) and Clerical Checking, and GATB Mark Making. For the girls, the loading on Disguised Words is higher than the

loadings on some of the spelling tests, and there are substantial loadings on more (but far from all) of the English tests and on three vocabulary tests, and also on the GATB Clerical and Form Matching tests.

The Coordination factor (5 for boys; 8 for girls) is essentially sensori-motor in nature rather than cognitive. In both groups it has loadings above .300 on Mark Making and the four apparatus-test scores, with no loading as high as .130 on any other test. The highest loadings are on the peg board tests, the next highest on Mark Making, and the lowest on Rivet Assembly and Rivet Disassembly. Even the rank orders of the five loadings are identical in the two groups.

These factor analyses are interesting perhaps as much for what was not found as for what was found. There should have been enough tests to separate an abstract-reasoning factor from the space factor, and to separate a mechanical-knowledge factor from both of them. The mathematics factor might have been separated from the computation factor, and some of the arithmetic reasoning and mathematical reasoning tests should have had substantial loadings, along with the abstract reasoning tests, on a reasoning factor separate from the space factor. The clerical factor might well have been separate from the perceptual factor defined by the identical-forms tests. Finally, the English language tests might have generated a factor separate from the spelling tests. These considerations lend some further weight to the suggestion that when a battery is assembled specifically for factor-analysis purposes, the tests should be more or less equally reliable, they should all be administered in consecutive half-day sessions, and for subgroups of the sample they sould be administered in different orders: ideally with the subgroups-by-orders design a latin square. The sample size should be large in comparison to the number of variables, and this disparity should increase as the reliabilities of the tests decrease.